

# 完全导电圆锥并矢 Green 函数\*\*\*

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**摘要** 本文运用算子法和并矢运算的分布理论法, 求解了完全导电圆锥并矢 Green 函数, 且证明解是满足电磁基本方程的.

**关键词** 电磁理论; 并矢 Green 函数; 完全导电圆锥

## 一、引 言

并矢 Green 函数是电磁理论中的一个重要概念, 广泛应用于求解天线和微波技术中的辐射和散射等问题<sup>[1,2]</sup>. 本文利用文献 [3—7] 的方法和技巧求解了完全导电圆锥并矢 Green 函数 (简称锥 DGF), 且证明解是满足并矢 Maxwell 方程和电流连续性方程的.

## 二、锥 DGF 的计算

为了求解电型和磁型锥 DGF, 引入矢量位型锥 DGF  $\bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}')$ , 它满足方程

$$\nabla^2 \bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}') + k^2 \bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}') = -\bar{\mathbf{I}}\delta(\mathbf{R} - \mathbf{R}') \quad (1)$$

在导电锥边界上,  $\hat{\mathbf{n}} \times \bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}') = 0$ ; 当  $\mathbf{R} \rightarrow \infty$  时,  $\bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}')$  满足 Sommerfeld 的辐射条件.

$\bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}')$  与电型和磁型锥 DGF  $\bar{\mathbf{G}}_e(\mathbf{R}/\mathbf{R}')$ ,  $\bar{\mathbf{G}}_m(\mathbf{R}/\mathbf{R}')$  之间的关系式是

$$\bar{\mathbf{G}}_e(\mathbf{R}/\mathbf{R}') = \left( \bar{\mathbf{I}} + \frac{1}{k^2} \nabla \nabla \right) \cdot \bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}') \quad (2a)$$

$$\bar{\mathbf{G}}_m(\mathbf{R}/\mathbf{R}') = \nabla \times \bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}') \quad (2b)$$

圆锥坐标系中的矢量波函数是<sup>[1]</sup>

$$\begin{aligned} \mathbf{L}_{e_{m\lambda}^\mu}(K) &= \nabla [\varphi_{e_{m\lambda}^\mu}(K)] \\ &= \left( \frac{\sqrt{\mu(\mu+1)}}{\sqrt{\lambda(\lambda+1)}} \right) \frac{j_\lambda^\mu(Kr)}{r} \mathbf{B}_{e_{m\lambda}^\mu}(\Omega) + \frac{dj_\lambda^\mu(Kr)}{dr} \mathbf{D}_{e_{m\lambda}^\mu}(\Omega) \end{aligned} \quad (3a)$$

$$\begin{aligned} \mathbf{M}_{e_{m\lambda}^\mu}(K) &= \nabla \times [\varphi_{e_{m\lambda}^\mu}(K) \hat{\mathbf{r}}] \\ &= \left( \frac{\sqrt{\mu(\mu+1)}}{\sqrt{\lambda(\lambda+1)}} \right) j_\lambda^\mu(Kr) \mathbf{C}_{e_{m\lambda}^\mu}(\Omega) \end{aligned} \quad (3b)$$

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$$\begin{aligned}
 \mathbf{N}_{\sigma_{m\lambda}^{\mu}}(K) &= \frac{1}{k} \nabla \times \nabla \times [\varphi_{\sigma_{m\lambda}^{\mu}}(K) \hat{\mathbf{r}}] \\
 &= \left( \frac{\sqrt{\mu(\mu+1)}}{\sqrt{\lambda(\lambda+1)}} \right) \left[ \frac{j_{\lambda}^{\mu}(Kr)}{Kr} + \frac{dj_{\lambda}^{\mu}(Kr)}{d(Kr)} \right] \mathbf{B}_{\sigma_{m\lambda}^{\mu}}(\Omega) \\
 &\quad + \left( \frac{\sqrt{\mu(\mu+1)}}{\sqrt{\lambda(\lambda+1)}} \right) \frac{j_{\lambda}^{\mu}(Kr)}{Kr} \mathbf{D}_{\sigma_{m\lambda}^{\mu}}(\Omega) \quad (3c)
 \end{aligned}$$

式中

$$\varphi_{\sigma_{m\lambda}^{\mu}}(k) = j_{\lambda}^{\mu}(Kr) P_{\lambda}^m(\cos\theta) \frac{\cos m\phi}{\sin m\phi} = j_{\lambda}^{\mu}(Kr) Y_{\sigma_{m\lambda}^{\mu}}(\Omega) \quad (4)$$

$$\mathbf{B}_{\sigma_{m\lambda}^{\mu}}(\Omega) = \frac{r}{\left( \frac{\sqrt{\mu(\mu+1)}}{\sqrt{\lambda(\lambda+1)}} \right)} \nabla [Y_{\sigma_{m\lambda}^{\mu}}(\Omega)] \quad (5a)$$

$$\mathbf{C}_{\sigma_{m\lambda}^{\mu}}(\Omega) = -\hat{\mathbf{r}} \times \mathbf{B}_{\sigma_{m\lambda}^{\mu}}(\Omega) \quad (5b)$$

$$\mathbf{D}_{\sigma_{m\lambda}^{\mu}}(\Omega) = \hat{\mathbf{r}} Y_{\sigma_{m\lambda}^{\mu}}(\Omega) \quad (5c)$$

式中,  $\hat{\mathbf{r}}$  是径向单位矢量,  $j_{\lambda}^{\mu}(Kr)$  是  $\mu$  或  $\lambda$  阶球 Bessel 函数,  $P_{\lambda}^m(\cos\theta)$  是  $m$  阶  $\mu$  或  $\lambda$  次连带 Legendre 函数。

由(3)–(5)式可知,就所涉及的函数结构条件而言,圆锥矢量波函数与球矢量波函数是相似的,因而我们可用类似的方法和步骤证明圆锥矢量本征函数  $\mathbf{L}$ ,  $\mathbf{M}$  和  $\mathbf{N}$  的正交性及归一化系数。

由边界条件,对单圆锥有

$$P_{\mu}^m(\cos\theta)|_{\theta=\theta_0} = 0 \quad (6a)$$

$$\frac{dP_{\lambda}^m(\cos\theta)}{d(\cos\theta)} \Big|_{\theta=\theta_0} = 0 \quad (6b)$$

对双圆锥有

$$P_{\mu}^m(\cos\theta)|_{\theta=\theta_1} = P_{\mu}^m(\cos\theta)|_{\theta=\theta_2} = 0 \quad (7a)$$

$$\frac{dP_{\lambda}^m(\cos\theta)}{d(\cos\theta)} \Big|_{\theta=\theta_1} = \frac{dP_{\lambda}^m(\cos\theta)}{d(\cos\theta)} \Big|_{\theta=\theta_2} = 0 \quad (7b)$$

式中  $\theta_0$ ,  $\theta_1$  和  $\theta_2$  为锥体的张角。

由并矢算子谱理论<sup>[4]</sup>,并矢波动方程(1)式的一般解为

$$\bar{\mathbf{U}}(\mathbf{R}/\mathbf{R}') = (\mathcal{L} - k^2)^{-1} \bar{\mathbf{F}}(\mathbf{R}/\mathbf{R}') = \sum_{\nu} \frac{\mathbf{U}_{\nu}(\mathbf{R}) \mathbf{A}_{\nu}(\mathbf{R}')}{\lambda_{\nu} - k^2} \quad (8)$$

式中,  $\Sigma$  表示对离散谱求和,对连续谱积分。

首先将(1)式中的广义函数  $\bar{\mathbf{I}}\delta(\mathbf{R} - \mathbf{R}')$  按圆锥矢量本征函数展开,得

$$\begin{aligned}
 \bar{\mathbf{I}}\delta(\mathbf{R} - \mathbf{R}') &= \int_0^{\infty} K^2 \sum_m \left\{ \sum_{\mu} C_{m\mu} \left[ \frac{\mu(\mu+1)}{K^2} \mathbf{L}_{\sigma_{m\mu}^{\mu}}(K) \mathbf{L}'_{\sigma_{m\mu}^{\mu}}(K) \right. \right. \\
 &\quad \left. \left. + \mathbf{N}_{\sigma_{m\mu}^{\mu}}(K) \mathbf{N}'_{\sigma_{m\mu}^{\mu}}(K) \right] + \sum_{\lambda} C_{m\lambda} \mathbf{M}_{\sigma_{m\lambda}^{\mu}}(K) \mathbf{M}'_{\sigma_{m\lambda}^{\mu}}(K) \right\} dK \quad (9)
 \end{aligned}$$

式中

$$C_{m\mu} = \frac{2 - \delta_0}{\pi^2 \mu(\mu + 1) I_{m\mu}} \quad (10a)$$

$$I_{m\mu} = \int_{\theta_0}^{\pi} [P_{\mu}^m(\cos \theta)]^2 \sin \theta d\theta$$

$$C_{m\lambda} = \frac{2 - \delta_0}{\pi^2 \lambda(\lambda + 1) I_{m\lambda}} \quad (10b)$$

$$I_{m\lambda} = \int_{\theta_0}^{\pi} [P_{\lambda}^m(\cos \theta)]^2 \sin \theta d\theta$$

式中

$$\delta_0 = \begin{cases} 0, & m \neq 0; \\ 1, & m = 0. \end{cases}$$

利用(8),(9)式,则可得到方程(1)的解为

$$\begin{aligned} \bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}') = & \int_0^{\infty} \frac{K^2}{K^2 - k^2} \sum_m \left\{ \sum_{\mu} C_{m\mu} \left[ \frac{\mu(\mu + 1)}{K^2} L_{\sigma_{m\mu}}(K) L'_{\sigma_{m\mu}}(K) \right. \right. \\ & \left. \left. + N_{\sigma_{m\mu}}(K) N'_{\sigma_{m\mu}}(K) \right] + \sum_{\lambda} C_{m\lambda} M_{\sigma_{m\lambda}}(K) M'_{\sigma_{m\lambda}}(K) \right\} dK \quad (11) \end{aligned}$$

由球 Bessel 函数的递推公式和留数定理,消去(11)式中对连续谱  $K$  的积分,得

$$\begin{aligned} \mathbf{G}_A(\mathbf{R}/\mathbf{R}') = & \sum_m \frac{i\pi k}{2} \left\{ \left[ \sum_{\mu} C_{m\mu} \left( \frac{\mu(\mu + 1)}{k^2} L_{\sigma_{m\mu}}^{(1)}(k) L'_{\sigma_{m\mu}}(k) \right. \right. \right. \\ & \left. \left. + N_{\sigma_{m\mu}}^{(1)}(k) N'_{\sigma_{m\mu}}(k) \right) + \sum_{\lambda} C_{m\lambda} M_{\sigma_{m\lambda}}^{(1)}(k) M'_{\sigma_{m\lambda}}(k) \right] \mu(r - r') \\ & \left. + \left[ \sum_{\mu} C_{m\mu} \left( \frac{\mu(\mu + 1)}{k^2} L_{\sigma_{m\mu}}(k) L'_{\sigma_{m\mu}}(k) + N_{\sigma_{m\mu}}(k) N'_{\sigma_{m\mu}}(k) \right) \right. \right. \\ & \left. \left. + \sum_{\lambda} C_{m\lambda} M_{\sigma_{m\lambda}}(k) M'_{\sigma_{m\lambda}}(k) \right] \mu(r' - r) \right\} \quad (12) \end{aligned}$$

式中矢量本征函数  $L_{\sigma_{m\mu}}^{(1)}(K)$ ,  $M_{\sigma_{m\lambda}}^{(1)}(k)$  和  $N_{\sigma_{m\mu}}^{(1)}(k)$  是用第一类球 Hankel 函数  $h_{\mu}^{(1)}(kr)$  替换(3)式中球 Bessel 函数  $j_{\mu}^{(1)}(Kr)$  得到;  $\mu(r - r')$  是 Heaviside 函数,定义为

$$\mu(r - r') = \begin{cases} 0, & r < r' \\ 1, & r \geq r' \end{cases}$$

由分布理论中的关系式  $\frac{d\mu(x - x')}{dx} = \delta(x - x')$  和广义函数的求导法则,我们有

$$\begin{aligned} & \frac{d}{dr} \left[ h_{\lambda}^{(1)}(kr) \frac{dj_{\lambda}^{\mu}(kr')}{dr'} \mu(r - r') + j_{\lambda}^{\mu}(kr) \frac{dh_{\lambda}^{(1)}(kr')}{dr'} \mu(r' - r) \right] \\ & = \frac{dh_{\lambda}^{(1)}(kr)}{dr} \frac{dj_{\lambda}^{\mu}(kr')}{dr'} \mu(r - r') + \frac{dj_{\lambda}^{\mu}(kr)}{dr} \frac{dh_{\lambda}^{(1)}(kr')}{dr'} \mu(r' - r) \\ & \quad + A\delta(r - r') \quad (13) \end{aligned}$$

式中

$$A = h_{\lambda}^{(1)}(kr) \frac{dj_{\lambda}^{(1)}(kr')}{dr'} - j_{\lambda}^{(1)}(kr) \frac{dh_{\lambda}^{(1)}(kr')}{dr'} \quad (14)$$

由球类 Bessel 函数  $\phi_l$  的递推关系式

$$l\phi_{l-1} - (l+1)\phi_{l+1} = (2l+1)\phi'_l \quad (15a)$$

和 Bessel 函数的关系式

$$Y_{n-1}J_n - Y_nJ_{n+1} = \frac{2}{\pi x} \quad (15b)$$

得

$$A = -\frac{i}{kr^2} \quad (16)$$

利用(13)式和矢量恒等式

$$\nabla \times [\varphi(r)\mathbf{A}(\mathbf{R})] = \varphi(r)\nabla \times \mathbf{A}(\mathbf{R}) + \hat{\mathbf{r}} \times \left[ \mathbf{A}(\mathbf{R}) \frac{d\varphi(r)}{dr} \right] \quad (17)$$

及

$$\nabla \nabla \cdot \mathbf{L}_{\sigma m \mu}(k) = -k^2 \mathbf{L}_{\sigma m \mu}(k) \quad (18a)$$

$$\nabla \times \mathbf{M}_{\sigma m \lambda}(k) = k \mathbf{N}_{\sigma m \lambda}(k) \quad (18b)$$

$$\nabla \times \mathbf{N}_{\sigma m \mu}(k) = k \mathbf{M}_{\sigma m \mu}(k) \quad (18c)$$

得

$$\begin{aligned} & \nabla \times [\mathbf{L}_{\sigma m \mu}^{(1)}(k) \mathbf{L}'_{\sigma m \mu}(k) \mu(r-r') + \mathbf{L}_{\sigma m \mu}(k) \mathbf{L}'_{\sigma m \mu}{}^{(1)}(k) \mu(r'-r)] \\ &= \sqrt{\mu(\mu+1)} \hat{\mathbf{r}} \times \frac{\mathbf{B}_{\sigma m \mu}(\Omega)}{r} \mathbf{D}_{\sigma m \mu}(\Omega') \left( \frac{-i}{kr^2} \right) \delta(r-r') \end{aligned} \quad (19a)$$

$$\begin{aligned} & \nabla \times \{ [\mathbf{M}_{\sigma m \lambda}^{(1)}(k) \mathbf{M}'_{\sigma m \lambda}(k) + \mathbf{N}_{\sigma m \mu}^{(1)}(k) \mathbf{N}'_{\sigma m \mu}(k)] \mu(r-r') \\ &+ [\mathbf{M}_{\sigma m \lambda}(k) \mathbf{M}'_{\sigma m \lambda}{}^{(1)}(k) + \mathbf{N}_{\sigma m \mu}(k) \mathbf{N}'_{\sigma m \mu}{}^{(1)}(k)] \mu(r'-r) \} \\ &= k \{ [\mathbf{N}_{\sigma m \lambda}^{(1)}(k) \mathbf{M}'_{\sigma m \lambda}(k) + \mathbf{M}_{\sigma m \mu}^{(1)}(k) \mathbf{N}'_{\sigma m \mu}(k)] \mu(r-r') \\ &+ \mathbf{N}_{\sigma m \lambda}(k) \mathbf{M}'_{\sigma m \lambda}{}^{(1)}(k) + \mathbf{M}_{\sigma m \mu}(k) \mathbf{N}'_{\sigma m \mu}{}^{(1)}(k) \} \mu(r'-r) \\ &+ (\mu(\mu+1))^{\frac{3}{2}} \hat{\mathbf{r}} \times \frac{\mathbf{B}_{\sigma m \mu}(\Omega)}{kr} \mathbf{D}_{\sigma m \mu}(\Omega') \left( \frac{i}{k^2 r^2} \right) \delta(r-r') \end{aligned} \quad (19b)$$

和

$$\begin{aligned} & \nabla \nabla \cdot [\mathbf{L}_{\sigma m \mu}^{(1)}(k) \mathbf{L}'_{\sigma m \mu}(k) \mu(r-r') + \mathbf{L}_{\sigma m \mu}(k) \mathbf{L}'_{\sigma m \mu}{}^{(1)}(k) \mu(r'-r)] \\ &= -k^2 [\mathbf{L}_{\sigma m \mu}^{(1)}(k) \mathbf{L}'_{\sigma m \mu}(k) \mu(r-r') + \mathbf{L}_{\sigma m \mu}(k) \mathbf{L}'_{\sigma m \mu}{}^{(1)}(k) \mu(r'-r)] \\ &+ \nabla \left[ \sqrt{\mu(\mu+1)} \hat{\mathbf{r}} \cdot \mathbf{D}_{\sigma m \mu}(\Omega) \frac{\mathbf{B}_{\sigma m \mu}(\Omega')}{r'} \left( \frac{i}{k^2 r^2} \right) \delta(r-r') \right] \\ &- k^2 \left( \frac{-i}{kr^2} \right) \mathbf{D}_{\sigma m \mu}(\Omega) \mathbf{D}_{\sigma m \mu}(\Omega') \delta(r-r') \end{aligned} \quad (20a)$$

$$\begin{aligned} & \nabla \nabla \cdot \{ [\mathbf{M}_{\sigma m \lambda}^{(1)}(k) \mathbf{M}'_{\sigma m \lambda}(k) + \mathbf{N}_{\sigma m \mu}^{(1)}(k) \mathbf{N}'_{\sigma m \mu}(k)] \mu(r-r') \\ & \quad + \mathbf{M}_{\sigma m \lambda}(k) \mathbf{M}'_{\sigma m \lambda}^{(1)}(k) + \mathbf{N}_{\sigma m \mu}(k) \mathbf{N}'_{\sigma m \mu}^{(1)}(k) \} \mu(r'-r) \} \\ & = \nabla \left[ (\mu(\mu+1))^{\frac{1}{2}} \mathbf{r} \cdot \mathbf{D}_{\sigma m \mu}(\Omega) \frac{\mathbf{B}_{\sigma m \mu}(\Omega')}{kr'} \left( \frac{-i}{k^2 r^2} \right) \delta(r-r') \right] \end{aligned} \quad (20b)$$

将  $\bar{\mathbf{G}}_A(\mathbf{R}/\mathbf{R}')$  代入(2)式,利用(19),(20)式得电型锥 DGF

$$\begin{aligned} \bar{\mathbf{G}}_e(\mathbf{R}/\mathbf{R}') & = \sum_m \frac{i\pi k}{2} \left\{ \left[ \sum_{\lambda} C_{m\lambda} \mathbf{M}_{\sigma m \lambda}^{(1)}(k) \mathbf{M}'_{\sigma m \lambda}(k) + \sum_{\mu} C_{m\mu} \mathbf{N}_{\sigma m \mu}^{(1)}(k) \mathbf{N}'_{\sigma m \mu}(k) \right] \mu(r-r') \right. \\ & \quad \left. + \left[ \sum_{\lambda} C_{m\lambda} \mathbf{M}_{\sigma m \lambda}(k) \mathbf{M}'_{\sigma m \lambda}^{(1)}(k) + \sum_{\mu} C_{m\mu} \mathbf{N}_{\sigma m \mu}(k) \mathbf{N}'_{\sigma m \mu}^{(1)}(k) \right] \mu(r'-r) \right\} \\ & \quad - \sum_m \sum_{\mu} \left( \frac{\pi C_{m\mu}}{2} \right) \frac{\mu(\mu+1)}{k^2 r^2} \mathbf{D}_{\sigma m \mu}(\Omega) \mathbf{D}_{\sigma m \mu}(\Omega') \delta(r-r') \end{aligned} \quad (21a)$$

和磁型锥 DGF

$$\begin{aligned} \bar{\mathbf{G}}_m(\mathbf{R}/\mathbf{R}') & = \sum_m \frac{i\pi k^2}{2} \left\{ \left[ \sum_{\mu} C_{m\mu} \mathbf{M}_{\sigma m \mu}^{(1)}(k) \mathbf{N}'_{\sigma m \mu}(k) \right. \right. \\ & \quad \left. + \sum_{\lambda} C_{m\lambda} \mathbf{N}_{\sigma m \lambda}^{(1)}(k) \mathbf{M}'_{\sigma m \lambda}(k) \right] \mu(r-r') \\ & \quad \left. + \left[ \sum_{\mu} C_{m\mu} \mathbf{M}_{\sigma m \mu}(k) \mathbf{N}'_{\sigma m \mu}^{(1)}(k) + \sum_{\lambda} C_{m\lambda} \mathbf{N}_{\sigma m \lambda}(k) \mathbf{M}'_{\sigma m \lambda}^{(1)}(k) \right] \mu(r'-r) \right\} \end{aligned} \quad (21b)$$

利用分布理论中的关系式

$$\sum_m \sum_{\mu} \frac{2-\delta_0}{2\pi I_{m\mu}} Y_{\sigma m \mu}(\Omega) Y_{\sigma m \mu}(\Omega') = \frac{\delta(\theta-\theta')\delta(\phi-\phi')}{\sin\theta} \quad (22)$$

则(21a)式中的具有  $\delta$  函数的奇异项可改写为

$$\bar{\mathbf{G}}_{eL}(\mathbf{R}/\mathbf{R}') = -\frac{1}{k^2} \hat{\mathbf{r}}\hat{\mathbf{r}} \frac{\delta(\theta-\theta')\delta(\phi-\phi')\delta(r-r')}{r^2 \sin\theta} \quad (23)$$

### 三、解的验证

对(21)式取旋度,利用(19)式得

$$\begin{aligned} \nabla \times \bar{\mathbf{G}}_e(\mathbf{R}/\mathbf{R}') & = \sum_m \frac{i\pi k^2}{2} \left\{ \left[ \sum_{\mu} C_{m\mu} \mathbf{M}_{\sigma m \mu}^{(1)}(k) \mathbf{N}'_{\sigma m \mu}(k) \right. \right. \\ & \quad \left. + \sum_{\lambda} C_{m\lambda} \mathbf{N}_{\sigma m \lambda}^{(1)}(k) \mathbf{M}'_{\sigma m \lambda}(k) \right] \mu(r-r') \\ & \quad + \left[ \sum_{\mu} C_{m\mu} \mathbf{M}_{\sigma m \mu}(k) \mathbf{N}'_{\sigma m \mu}(k) \right. \\ & \quad \left. + \sum_{\lambda} C_{m\lambda} \mathbf{N}_{\sigma m \lambda}(k) \mathbf{M}'_{\sigma m \lambda}^{(1)}(k) \right] \mu(r'-r) \left. \right\} \end{aligned}$$

$$\begin{aligned}
& + (\mu(\mu+1))^{3/2} \hat{r} \times \frac{B_{\sigma_{m\mu}}(\Omega)}{kr} D_{\sigma_{m\mu}}(\Omega') \left( \frac{i}{k^2 r^2} \right) \delta(r-r') \\
& - \frac{\mu(\mu+1)}{k^2 r^2} \nabla \times D_{\sigma_{m\mu}}(\Omega) D_{\sigma_{m\mu}}(\Omega') \delta(r-r') \\
& = \bar{G}_m(\mathbf{R}/\mathbf{R}')
\end{aligned} \tag{24a}$$

和

$$\begin{aligned}
\nabla \times \mathbf{G}_m(\mathbf{R}/\mathbf{R}') & = \nabla \times \nabla \times \bar{G}_A(\mathbf{R}/\mathbf{R}') = (-\nabla^2 + \nabla \nabla \cdot) \bar{G}_A(\mathbf{R}/\mathbf{R}') \\
& = [k^2 \bar{G}_A(\mathbf{R}/\mathbf{R}') + I \delta(\mathbf{R} - \mathbf{R}')] \\
& \quad + \sum_m \sum_\mu \left( \frac{-i\pi k}{2} \right) [\mu(\mu+1) (L_{\sigma_{m\mu}}^{(1)}(k) L'_{\sigma_{m\mu}}(k) \mu(r-r') \\
& \quad + L_{\sigma_{m\mu}}(k) L'_{\sigma_{m\mu}}(k) \mu(r'-r))] - \hat{r} \hat{r} \delta(\mathbf{R} - \mathbf{R}') \\
& = \sum_m \frac{i\pi k^3}{2} \left\{ \left[ \sum_\lambda C_{m\lambda} M_{\sigma_{m\lambda}}^{(1)}(k) M'_{\sigma_{m\lambda}}(k) \right. \right. \\
& \quad \left. \left. + \sum_\mu C_{m\mu} N_{\sigma_{m\mu}}^{(1)}(k) N'_{\sigma_{m\mu}}(k) \right] \mu(r-r') \right. \\
& \quad \left. + \left[ \sum_\lambda C_{m\lambda} M_{\sigma_{m\lambda}}(k) M'_{\sigma_{m\lambda}}(k) \right. \right. \\
& \quad \left. \left. + \sum_\mu C_{m\mu} N_{\sigma_{m\mu}}(k) N'_{\sigma_{m\mu}}(k) \right] \mu(r'-r) \right\} \\
& \quad - \hat{r} \hat{r} \delta(\mathbf{R} - \mathbf{R}') + I \delta(\mathbf{R} - \mathbf{R}') \\
& = k^2 \bar{G}_c(\mathbf{R}/\mathbf{R}') + I \delta(\mathbf{R} - \mathbf{R}')
\end{aligned} \tag{24b}$$

对 (21a) 式取散度, 利用 (20) 式得

$$\begin{aligned}
\nabla \cdot \bar{G}_c(\mathbf{R}/\mathbf{R}') & = - \sum_m \sum_\mu \frac{i\pi k}{2} \mu(\mu+1) \hat{r} \cdot D_{\sigma_{m\mu}}(\Omega) B_{\sigma_{m\mu}}(\Omega') \left( \frac{i}{k^2 r^2} \right) \delta(r-r') \\
& \quad - \nabla \cdot \frac{1}{k^2} \hat{r} \hat{r} \delta(\mathbf{R} - \mathbf{R}') \\
& = - \frac{1}{k^2} \nabla \cdot \bar{I}_i \delta(\mathbf{R} - \mathbf{R}') - \frac{1}{k^2} \nabla \cdot \hat{r} \hat{r} \delta(\mathbf{R} - \mathbf{R}') \\
& = - \frac{1}{k^2} \nabla \cdot \bar{I} \delta(\mathbf{R} - \mathbf{R}')
\end{aligned} \tag{25}$$

(24)和(25)式表明本文给出的电型和磁型锥 DGF 是满足并矢 Maxwell 方程和电流连续性原理的。

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## DYADIC GREEN'S FUNCTIONS FOR PERFECTLY CONDUCTING CIRCULAR CONES

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**Abstract** The dyadic Green's functions for perfectly conducting circular cones are derived by means of the operator method and the theory of distribution of dyadic operation. It is shown that the electric and magnetic dyadic Green's functions given satisfy the basic dyadic equations.

**Key words** Electromagnetic field theory; Perfectly conducting circular cone; Dyadic Green's function